

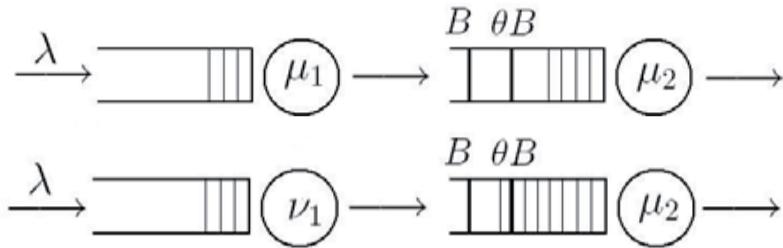
Efficient Simulation of a Tandem Queue with Server Slow-down

Denis Miretskiy, Werner Scheinhardt (University of Twente), Michel Mandjes (University of Amsterdam)

{d.miretskiy, w.r.w.scheinhardt}@math.utwente.nl

mmandjes@science.uva.nl

Model



Network consist of two M/M/1 queues;
 first queue: infinite buffer; service rate μ_1 if the number of jobs in the second buffer is less then θB , $\theta \leq 1$ and ν_1 otherwise ($\nu_1 < \mu_1$)
 second queue: buffer of finite capacity B ; service rate μ_2

The Slow-down property of the first server (speed policy based on the content of the second queue) protects second buffer from a frequent overflow.

Problem

The main target of our current research is the probability of overflow in the second buffer of the Slow-down (and Jackson) network during a busy cycle.

Definition. Busy cycle is the time between two successive epochs at which the process leaves the empty state.

$$p_B := \mathbb{P}_{(1,0)}(X_2(T_B) = B)$$

$$T_B = \min\{t > 0 | X_2(t) = B \text{ or } (X_1(t), X_2(t)) = (0, 0)\}$$

Theorem 1. If $\theta = 1$, then the overflow probability p_B is asymptotically geometrical in B with parameter $\rho_2 = \frac{\lambda}{\mu_2}$, more precisely:

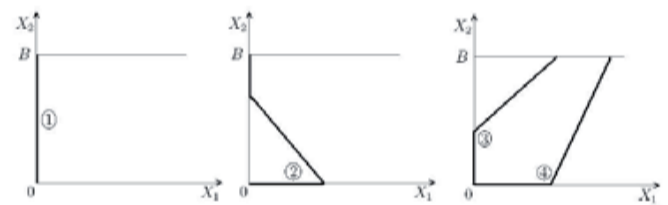
$$\lim_{B \rightarrow \infty} \frac{1}{B} \log(p_B) = \log(\rho_2)$$

Main Conjecture

In this model (and some other more general models), the typical path leading to the rare event consists of a finite concatenation of a straight lines.

Benefit: the solution boils down to optimizing over a finite number of possible path-types, i.e., we reduced the problem to a combinatorial problem.

Path-types ($\theta = 1$)



Importance Sampling

The idea of important sampling is simulating the system under “artificial” measure which guaranties frequent occurrence of rare event.

We define heuristic cost-function $I(\tilde{\lambda} | \lambda) = \lambda - \tilde{\lambda} + \tilde{\lambda} \log(\frac{\tilde{\lambda}}{\lambda})$ based on Kullback-Leibler distance. Function $\frac{I(\tilde{\lambda} | \tilde{\lambda}) + I(\tilde{\mu}_1 | \mu_1) + I(\tilde{\mu}_2 | \mu_2)}{\tilde{\mu}_1 - \tilde{\mu}_2}$ represents the total “cost” of the path per vertical unit. Minimizing it over all “tilded” parameters we will find the optimal “new” measure and the most likely path to overflow in the second buffer.

Results

Case $\lambda < \mu_1 < \mu_2$, $\theta = 1$

optimal measure:

$$(\tilde{\lambda}, \tilde{\mu}_1, \tilde{\mu}_2) = (\mu, \lambda, \mu_2) \text{ until } X_1(t) = -\alpha^{-1}B$$

$$(\tilde{\lambda}, \tilde{\mu}_1, \tilde{\mu}_2) = (\mu_1, \mu_2, \lambda) \text{ afterwards}$$

optimal path:



overflow probability:

$$(\lambda = 0.13, \mu_1 = 0.17, \mu_2 = 0.7)$$

$$B = 20: 7.5 \cdot 10^{-15} \pm 1.2 \cdot 10^{-15}$$

$$B = 100: 1.73 \cdot 10^{-73} \pm 1.73 \cdot 10^{-74}$$

Case $\lambda < \nu_1 < \mu_2 < \mu_1$, $\theta < 1$

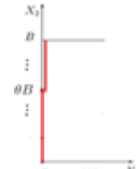
optimal measure:

$$(\tilde{\lambda}, \tilde{\mu}_1, \tilde{\mu}_2) = (\mu_2, \mu_1, \lambda) \text{ below } \theta B$$

$$(\tilde{\lambda}, \tilde{\nu}_1, \tilde{\mu}_2) = (\sqrt{\lambda \nu_1 / z}, \sqrt{\lambda \nu_1 / z}, \mu_2 z) \text{ above } \theta B$$

$$\lambda + \nu_1 + \mu_2(1 - z) = 2\sqrt{\frac{\lambda \nu_1}{z}}, z \in (0, 1)$$

optimal path:



overflow probability:

$$(\lambda = 0.1, \mu_1 = 0.7, \nu_1 = 0.15, \mu_2 = 0.2),$$

$$\theta = 0.8$$

$$B = 20: 3.79 \cdot 10^{-7} \pm 1.09 \cdot 10^{-9}$$

$$B = 100: 3.54 \cdot 10^{-32} \pm 1.89 \cdot 10^{-34}$$

Conclusions:

1. Our heuristics applicable in time irreversible systems.
2. Optimal “artificial” measure provides a great speed-up of simulations.
3. Optimal “new” measure is direct result of using our heuristics.

Future research:

1. Searching for better state-independent (or state-dependent) “new” measures.
2. Proving analogue of the *Theorem 1* for the case $0 < \theta < 1$
3. Further research of systems with server slow-down.

Acknowledgment:

Part of this research has been funded by the Dutch BSIK/BRICKS project PDC2